MA 552, Spring 2011

List of topics.

For the final, you are supposed to know notions, statements and their proofs covered in the list below, and be able to use these notions and statements, as well as ideas contained in their proofs, to solve problems.

HK stands for Kenneth Hoffman, Ray Kunze's textbook. Hefferon stands for Jim Hefferon's textbook.

(1) **Linear equations:** fields, linear equations, matrix of a linear system, elementary row operations, equivalent systems, row-reduced form, row-reduced echelon form, solving system of linear equations, rank of a matrix/system (HK 1.1–1.4).
(2) **Vector spaces basics:** fields, vector space, linear combinations, bases, dimension, finite/infinite dimensional spaces, coordinates, change of coordinates, transition matrix(HK 2.1–2.4), Lagrange interpolation (HK 4.3), direct sums (HK 6.6).
(3) **Relation between linear equations and vector spaces:** space of solutions of a homogenous system, constructing system whose solution space coincides with a given subspace, connection between coordinates in a subspace and in the ambient space (HK 2.5–2.6).
(4) **Linear transformations:** definition, matrix of a linear transformation, range (image) and null space (kernel) of a linear transformation, rank plus nullity, equality of row rank and column rank of a matrix, connection between matrix multiplication and composition of linear transformations, change of basis, similar matrices, invertible matrices, equivalent conditions for invertibility, finding inverse matrix, isomorphism (HK 3.1–3.4, 1.5, 1.6), projections (HK 6.6).
(5) **Determinant:** multilinear functions, alternating functions, determinant as a volume (multilinear alternating function), existence and uniqueness of determinant, permutation formula for determinant, computing determinants, determinant of block-diagonal and triangular matrices, classical adjoint, row/column expansion of determinant, fake expansion, explicit formula for inverse matrix, equivalent conditions for invertibility of a matrix, Cramer's rule (HK 5.2–5.4).
(6) **Dual space:** linear functionals, hyperspaces, isomorphism of a finite dimensional space to its dual space, dual basis, annihilator of a set, dimension of annihilator, double dual, canonical isomorphism between finite dimensional space and its double dual, transpose of a linear transformation (HK 3.5–3.7).
(7) **Theory of a single linear operator, characteristic values:** characteristic values, characteristic vectors, characteristic polynomial, Cayley–Hamilton theorem, diagonalization (HK 6.1–6.2, or Hefferon Chapter Five, II).
(8) **Theory of a single linear operator, the Jordan form:** generalized range and null space, nilpotent operators, string basis for a nilpotent operator, invariant subspaces, generalized range space decomposition, Jordan form theorem, finding dimensions of Jordan blocks, computing Jordan form of a matrix and finding an appropriate basis, raising matrix to a power using Jordan form, finding minimal polynomial of a matrix using Jordan form (Lectures, Hefferon Chapter Five, III, IV, or certain portions of HK 6.3–7.3).
(9) **Extra topics** (not included in final): Linear error correcting codes and Hamming code (extra topic problem set, or assorted sources). Google PageRank, Markov chains and approximating first characteristic vector and value using method of powers (lectures, or assorted sources).